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AN UNSTEADY PULSATILE FLOW OF SOME MHD NON-NEWTONIAN NANOFLUIDS WITH HALL CURRENT AND ION SLIP THROUGH A POROUS MEDIUM

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ABSTRACT

The current study examines the pulsatile flow of MHD Non-Newtonian nanofluids with hall current, ion slip and thermal radiation. Using regular perturbation technique, velocity and unsteady temperature distributions for both Eyring-Powell nanofluid and Casson nano fluid are analysed by suitable transformations. When we compare Eyring-Powell nanofluid model with Casson nano fluid model it is ob that u is high for Casson nanofluid model, and Eyring-Powell nano fluid shows the highest temperature variations.

KEYWORDS*: *Pulsatile Flow, Eyring-Powell Nanofluid, Casson Nanofluid, Hall Current & Ion Slip and Thermal Radiation

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Original Article

INTRODUCTION

The study of Non-Newtonian fluids has a vital role in fields like food processing, polymer processing, etc. Hayat et al. [1] Nadeem and Saleem[2], Tasawar Hayat et al. [3], Macharla Jayachandra Babu et al. [4], Vijayalakshmi and Shankar Bandari. [5], Asha and Sunitha[6], Mallick and Misra [7] and Misra and Sinha [8] are some of the researchers who have focused on MHD Eyring-Powell Nanofluid in recent years. A few recent investigators that may be mentioned are Motsa and Shateyi[9], Shehzad et al.[10], Rizwanulhaq et al. [11], Mustafa and Junaid Ahmad Khan [12], Imran Ullah et al. [13], Sreekala and Kesavareddy [14], Kamran et al. [15], Rajib Biswas and Sardar Firoz Ahmmed [16], Pudhari srilatha[17], Jannath Begam et al.[18], Deivanayaki et al.[19], Jannath Begam et al. [20]and Jannath Begam et al. [21].

Here, we attempt to examine the hall current and ion slip effects on pulsatile flow of some Non-Newtonian fluids with thermal radiation.

MATHEMATICAL FORMULATION

The constitutive equation for Eyring-Powell nanofluid and Casson nanofluid is

$$\tau_{ij} = \mu_{nf}\frac{\partial V_i}{\partial x_j} + \frac{1}{\beta}\sin h^{-1}\left(\frac{1}{c}\frac{\partial V_i}{\partial x_j}\right) \tag{1}$$

Since $\sin h^{-1} x \approx x$, $|x| << 1$, then

$$\tau_{ij} = \mu_{nf}\frac{\partial V_i}{\partial x_j} + \frac{1}{\beta}\left(\frac{1}{c}\frac{\partial V_i}{\partial x_j}\right)$$

$$= \mu_{nf}\left(1 + \frac{1}{\beta c \mu_{nf}}\right)\frac{\partial V_i}{\partial x_j} \tag{2}$$

$$\tau_{ij} = \begin{cases} 2\left(\mu B + \frac{P_{y^*}}{\sqrt{2\pi}}\right) e_{ij}, \pi > \pi_c \\ 2\left(\mu B + \frac{P_{y^*}}{\sqrt{2\pi_c}}\right) e_{ij}, \pi < \pi_c \end{cases}$$

$$\tau_{ij} = \mu_{nf}\left(1 + \frac{1}{\beta}\right)\frac{\partial V_i}{\partial x_j} \tag{3}$$

Unsteady incompressible Eyring-Powell nanofluid and Casson nanofluid are examined, with its x-axis selected as lower plate and transversely applied magnetic field along y axis. Using Boussinesq approximation, we have

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho_{nf}}\frac{\partial P^*}{\partial x^*} + q\vartheta_{nf}\left(1 + \frac{1}{\beta c \mu_{nf}}\right)\frac{\partial^2 u^*}{\partial y^{*2}} + (1-q)q\vartheta_{nf}\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\mu_{nf}}{\rho_{nf}}u^* + \frac{\sigma_{nf}B_0^2(1+\beta_e\beta_i)}{\rho_{nf}[(1+\beta_e\beta_i)^2+\beta_e^2]}u^* \tag{4}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k_{nf}}{(\rho C_P)_{nf}}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{1}{(\rho C_P)_{nf}}\frac{\partial q_r}{\partial y^*} + q\frac{\mu_{nf}\left(1+\frac{1}{\beta c \mu_{nf}}\right)}{(\rho C_P)_{nf}}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + (1-q)\frac{\mu_{nf}\left(1+\frac{1}{\beta}\right)}{(\rho C_P)_{nf}}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{Q_0}{(\rho C_P)_{nf}}(T^* - T_0) \tag{5}$$

In equations (4) and (5), $q\epsilon[0,1]$, it states Eyring-Powell nanofluid for q=1 and Casson nanofluid for q=0.

and

$$u^* = 0, T^* = T_0 at y^* = 0 \tag{6}$$

$$u^* = 0, T^* = T_1 at y^* = h \tag{7}$$

The physical properties of nanofluid such as $\mu_{nf}, \rho_{nf}, (\rho C_P)_{nf}$ and k_{nf} are given as

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{8}$$

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \tag{9}$$

$$(\rho C_P)_{nf} = (1-\phi)(\rho C_P)_f + \phi(\rho C_P)_s \tag{10}$$

$$\frac{k_{nf}}{k_f} = \frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)} \tag{11}$$

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^{*4}}{\partial y^*} \tag{12}$$

$$T^{*4} \cong 4T_1^3 T^* - 3T_1^4 \tag{13}$$

Using Equations(12) and (13), Equation (5) becomes

$$\frac{\partial T^*}{\partial t^*} = \frac{k_{nf}}{(\rho C_P)_{nf}}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{1}{(\rho C_P)_{nf}}\left[\frac{16\sigma^*}{3k^*}T_1^3\frac{\partial^2 T^*}{\partial y^{*2}}\right] + q\frac{\mu_{nf}\left(1+\frac{1}{\beta c \mu_{nf}}\right)}{(\rho C_P)_{nf}}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + (1-q)\frac{\mu_{nf}\left(1+\frac{1}{\beta}\right)}{(\rho C_P)_{nf}}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{Q_0}{(\rho C_P)_{nf}}(T^* - T_0) \tag{14}$$

$$-\frac{1}{\rho_f}\frac{\partial P^*}{\partial x^*} = A(1 + \epsilon e^{iwt}) \tag{15}$$

Introducing the following dimensionless variables

$$u = \frac{u^{*}\omega}{A}, t = t^{*}\omega, x = \frac{x^{*}}{h}, y = \frac{y^{*}}{h}, \theta = \frac{T^{*}-T_0}{T_1-T_0}, P = \frac{P^{*}}{A_{\rho_f}h}, k_0 = \frac{1}{\beta c\mu_f} \tag{16}$$

And parameters as in Equations (8) to (11) into Equations (15),(4) and (14), we get

$$-\frac{\partial P}{\partial x} = 1 + \epsilon e^{it} \tag{17}$$

$$\frac{\partial u}{\partial t} = -\frac{1}{A_1}\frac{\partial P}{\partial x} + q\frac{A_2}{A_1}\left(1+\frac{k_0}{A_2}\right)\frac{1}{R}\frac{\partial^2 u}{\partial y^2} + (1-q)\frac{A_2}{A_1}\left(1+\frac{1}{\beta}\right)\frac{1}{R}\frac{\partial^2 u}{\partial y^2} - \frac{A_2}{A_1}\left(\frac{1}{RD_a} + M_1\right)u \tag{18}$$

$$\frac{\partial \theta}{\partial t} = \left(\frac{A_4}{A_3} + \frac{4Rd}{3A_3}\right)\frac{1}{RP_r}\frac{\partial^2\theta}{\partial y^2} + q\frac{A_2}{A_3}\left(1+\frac{k_0}{A_2}\right)\frac{Ec}{R}\left(\frac{\partial u}{\partial y}\right)^2 + (1-q)\frac{A_2}{A_3}\left(1+\frac{1}{\beta}\right)\frac{Ec}{R}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q}{A_3 R}\theta \tag{19}$$

and

$$u = 0, \theta = 0 \ at y = 0 \tag{20}$$

$$u = 0, \theta = 1 \ at y = 1 \tag{21}$$

where $A_1 = (1-\phi) + \phi\frac{\rho_s}{\rho_f}$, $A_2 = \frac{1}{(1-\phi)^{2.5}}$,

$A_3 = (1-\phi) + \phi\frac{(\rho C_P)_s}{(\rho C_P)_f}$, $A_4 = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}$,

$Pr = \frac{(\rho C_P)_f v_f}{k_f}$, $Ec = \frac{A^2}{\omega^2(\rho C_P)_f(T_1 - T_0)}$, $R = \frac{\omega h^2}{\mu_f}$, $k_0 = \frac{1}{\beta c\mu_f}$, $M = \frac{\sigma_{nf}B_0^2}{\rho_{nf}}$

$Da = \frac{k}{h^2}$, $Rd = \frac{4\sigma^{*}T_1^3}{k_f k^{*}}$, $Q = \frac{Q_0 h^2}{(\rho C_P)_f \mu_f}$, $M_1 = \frac{M(1+\beta_e\beta_i)}{(1+\beta_e\beta_i)^2 + \beta_e^2}$

SOLUTION FOR THE PROBLEM

The u and θ are written as

$$u = u_0(y) + \epsilon u_1(y)e^{it} \tag{22}$$

$$\theta = \theta_0(y) + \epsilon\theta_1(y)e^{it} + \epsilon^2\theta_2(y)e^{2it} \tag{23}$$

Using Equations (17), (22) and (23) in Equations (18) and (19) and comparing the terms of like powers of ϵ, we get

$$q\frac{A_2}{A_1}\left(1+\frac{k_0}{A_2}\right)\frac{1}{R}u_0'' + (1-q)\frac{A_2}{A_1}\left(1+\frac{1}{\beta}\right)\frac{1}{R}u_0'' - \frac{A_2}{A_1}\left(\frac{1}{RD_a} + M_1\right)u_0 = -\frac{1}{A_1} \tag{24}$$

$$q\frac{A_2}{A_1}\left(1+\frac{k_0}{A_2}\right)\frac{1}{R}u_1'' + (1-q)\frac{A_2}{A_1}\left(1+\frac{1}{\beta}\right)\frac{1}{R}u_1'' - \frac{A_2}{A_1}\left(\frac{1}{RD_a} + M_1\right)u_1 - iu_1 = -\frac{1}{A_1} \tag{25}$$

$$\left(\frac{A_4}{A_3} + \frac{4Rd}{3A_3}\right)\frac{1}{RP_r}\theta_0'' + q\frac{A_2}{A_3}\left(1+\frac{k_0}{A_2}\right)(u_0')^2 + (1-q)\frac{A_2}{A_3}\left(1+\frac{1}{\beta}\right)(u_0')^2 + \frac{Q}{A_3 R}\theta_0 = 0 \tag{26}$$

$$\left(\frac{A_4}{A_3} + \frac{4Rd}{3A_3}\right)\frac{1}{RP_r}\theta_1'' + 2q\frac{A_2}{A_3}\left(1+\frac{k_0}{A_2}\right)u_0'u_1' + 2(1-q)\frac{A_2}{A_3}\left(1+\frac{1}{\beta}\right)u_0'u_1'\frac{Q}{A_3 R}\theta_1 - i\theta_1 = 0 \tag{27}$$

$$\left(\frac{A_4}{A_3} + \frac{4Rd}{3A_3}\right)\frac{1}{RP_r}\theta_2'' + 2q\frac{A_2}{A_3}\left(1+\frac{k_0}{A_2}\right)(u_1')^2 + 2(1-q)\frac{A_2}{A_3}\left(1+\frac{1}{\beta}\right)(u_1')^2 + \frac{Q}{A_3 R}\theta_2 - 2i\theta_2 = 0 \tag{28}$$

and

$$u_0 = 0, \theta_0 = 0 \text{ at } y = 0 \tag{29}$$

$$= 0, \theta_0 = 1 \text{ at } y = 1 \tag{30}$$

$$u_1 = 0, \theta_1 = 0, \theta_2 = 0 \text{ at } y = 0 \tag{31}$$

$$u_1 = 0, \theta_1 = 0, \theta_2 = 0 \; at y = 1 \tag{32}$$

By solving equations (24) to (28) with the corresponding boundary conditions (29) to (32), we obtain

$$u_0 = B_7 e^{\sqrt{B_3 y}} + B_6 e^{-\sqrt{B_3 y}} - \frac{B_4}{B_3} \tag{33}$$

$$u_1 = B_9 e^{\sqrt{B_5 y}} + B_8 e^{-\sqrt{B_5 y}} - \frac{B_4}{B_5} \tag{34}$$

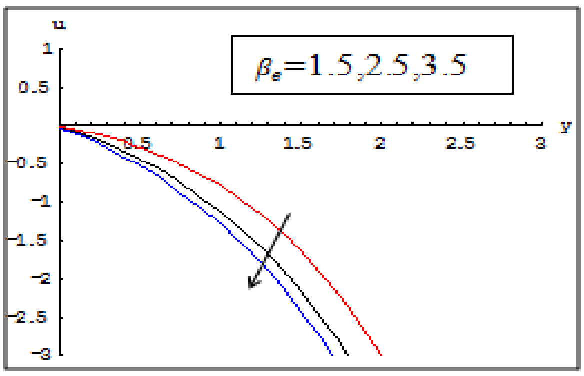
$$\theta_0 = c_{11} \cos \sqrt{d_2}\, y + c_{12} \sin \sqrt{d_2}\, y + c_8 e^{2\sqrt{B_3 y}} + c_9 e^{-2\sqrt{B_3 y}} + c_{10} \tag{35}$$

$$\theta_1 = D_6 e^{\sqrt{C_4 y}} + D_5 e^{-\sqrt{C_4 y}} + D_1 e^{m_1 y} + D_2 e^{m_2 y} + D_3 e^{m_3 y} + D_4 e^{m_4 y} \tag{36}$$

$$\theta_2 = D_{11} e^{\sqrt{C_6 y}} + D_{10} e^{-\sqrt{C_6 y}} + D_7 e^{2\sqrt{B_5 y}} + D_8 e^{-2\sqrt{B_5 y}} + D_9 \tag{37}$$

RESULTS AND DISCUSSIONS

To find the physical insight, the velocity and temperature profiles are studied for different parameters and discussed graphically. Figure (1a) pictures, velocity diminishes for k_0 and the opposite behavior has keenly been observed from figure (1b) for the Casson nanofluid parameter for different β_e. Velocity diminishes for ion slip parameter β_i as illustrated in figure(2a). The contrasting effect has been pointed out in figure (2b). Velocity diminishes as k_0 and β increase as examined in figure (3a) and (3b). Casson nanofluid shows little high increase in unsteady temperature for various Ec than k_0 as analysed in figure(4). β shows slightly increasing θ_t than k_0 for various heat source parameter Q is observed from Figure (5).



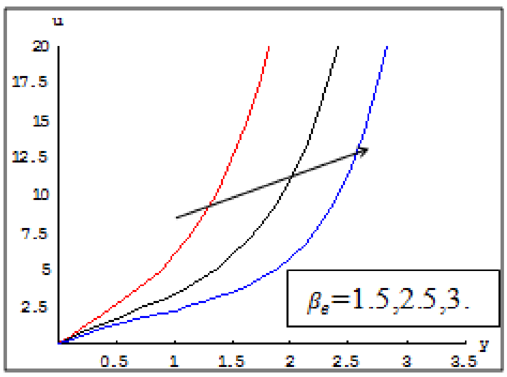


Figure (1a): Impact of β_e on Velocity Profile u of of Eyring-Powell Nanofluid **Figure (1b): Impact of β_e on Velocity Profile u Cass on Nanofluid**

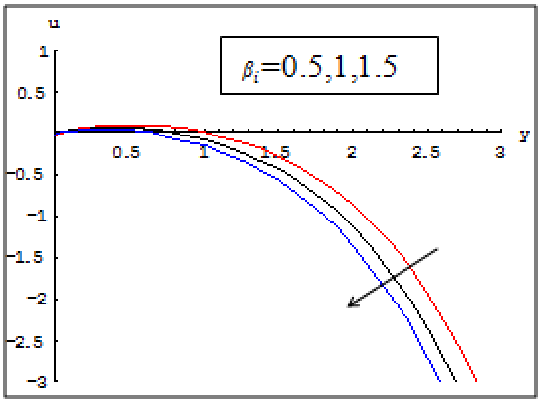


Figure (2a): Impact of β_i on Velocity Profile u of Eyring-Powell Nanofluid

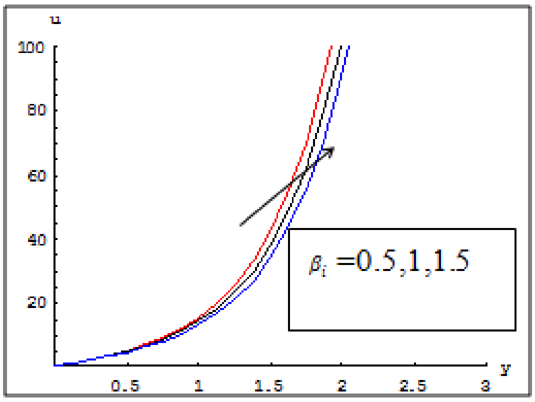


Figure (2b): Impact of $\beta_i$on Velocity Profile u of Cass on Nanofluid

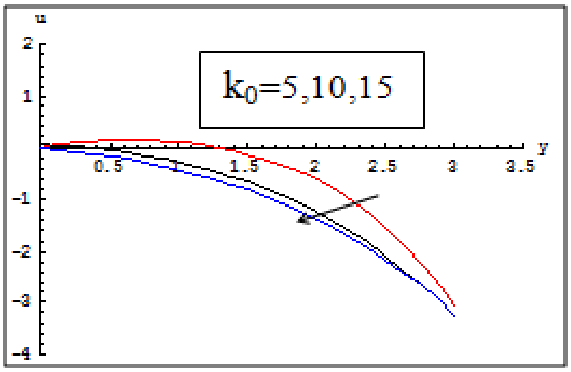


Figure (3a): Impact of k_0 on Velocity Profile u of Eyring-Powell Nanofluid

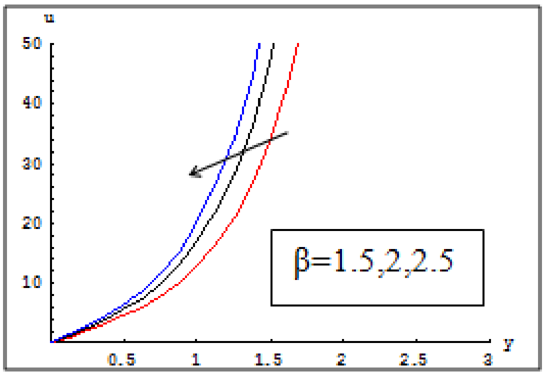


Figure (3b): Impact of β on Velocity Profile u of Cass on Nanofluid

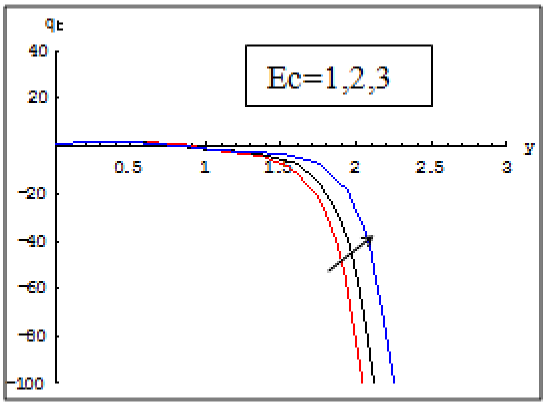


Figure (4a): Impact of Ec on Unsteady Temperature Profile θ_t of Eyring-Powell Nanofluid

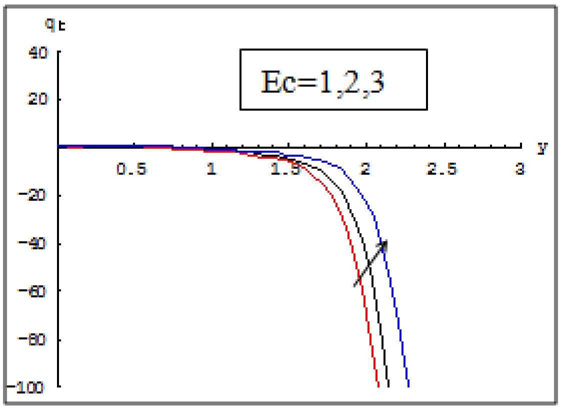


Figure (4b): Impact of Ec on Unsteady Temperature Profile θ_t of Cass on Nanofluid

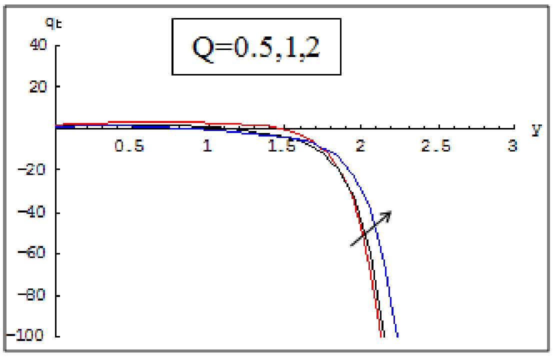


Figure (5a): Impact of Q on Unsteady Temperature Profile θ_t of Eyring-Powell Nanofluid

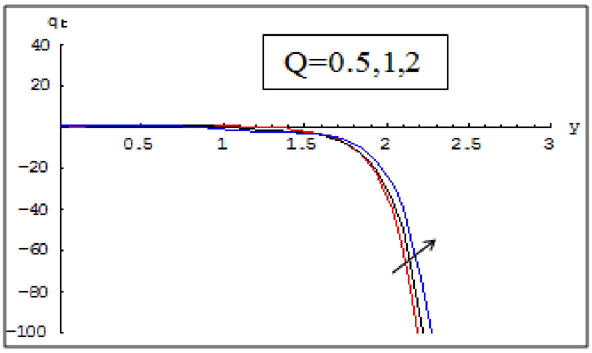


Figure (5b): Impact of Q on Unsteady Temperature Profile θ_t of Cass on Nanofluid

CONCLUSIONS

Finally, we conclude that,

- The velocity profile decreases with Hall current and ion slip effect for k_0 and increase for β.
- The velocity profile gets decreased with increasing values of k_0 and β, and the temperature profile gets increased with enhanced Ec and Q for both k_0 and β.

Although a very slight difference is observed between k_0 and β, we conclude that k_0 shows minute variations when compared with β.

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APPENDIX

NOMENCLATURE

β	Casson nanofluid parameter
Q_0	Coefficient of Heat source/Sink
Da	Darcy Number
ρ_{nf}	Density of Nanofluid
ρ_f	Density of the Base fluid
ρ_s	Density of the Nanoparticle
h	Distance Between the plates
μ_{nf}	Dynamic viscosity of the Nanofluid
Ec	Eckert Number
$(\rho C_P)_{nf}$	Effective Heat Capacitance of Nanofluid
k_0	Eyring-Powell nanofluid parameter
R	Frequency parameter
β_e	Hall current parameter
$(\rho C_P)_f$	Heat Capacitance of Base fluid
$(\rho C_P)_s$	Heat Capacitance of Nanoparticle
Q	Heat Source parameter
β_i	Ion slip parameter
k^*	Mean Absorption Coefficient
ϕ	Nanoparticles volume fraction
k	Permeability of porous medium
Pr	Prandtl number
P*	Pressure
Rd	Radiation Parameter
q_r	Radiative Heat flux
σ^*	Stefan-Boltzmann Constant
k_f	Thermal conductivity of Base fluid
k_{nf}	Thermal conductivity of Nanofluid
k_s	Thermal conductivity of Nanoparticle
u*	Velocity component in x*-direction
μ_f	Viscosity of the Base fluid